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AN h^4 ACCURATE VORTICITY–VELOCITY FORMULATION FOR CALCULATING FLOW PAST A CYLINDER

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SUMMARY

A method of solution for the two-dimensional Navier–Stokes equations for incompressible flow past a cylinder is given in which the equation of continuity is solved by a step-by-step integration procedure at each stage of an iterative process. Thus the formulation involves the solution of one first-order and one second-order equation for the velocity components, together with the vorticity transport equation. The equations are solved numerically by h^4-accurate methods in the case of steady flow past a circular cylinder in the Reynolds number range 10–100. Results are in satisfactory agreement with recent h^4-accurate calculations. An improved approximation to the boundary conditions at large distance is also considered.

KEY WORDS: Navier–Stokes equations; vorticity–velocity formulation; finite difference methods

1. INTRODUCTION

In recent times there has been interest in the vorticity–velocity formulation for solving the Navier–Stokes equations as an alternative to the primitive variable method and, in two dimensions, to the vorticity–stream function approach. The basic principle of the formulation is to solve in some manner or other the equations which express the velocity components in terms of vorticity components, together with the set of equations which govern the vorticity components. At the same time the equation of continuity must be satisfied. For an incompressible fluid this equation is

$$\operatorname{div} \mathbf{v} = 0, \tag{1}$$

where $\mathbf{v}$ is the velocity vector. The vorticity vector $\boldsymbol{\omega}$ is defined by

$$\boldsymbol{\omega} = \operatorname{curl} \mathbf{v} \tag{2}$$

and it is easily deduced from the Navier–Stokes equations for incompressible fluids that

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + (\mathbf{v} \cdot \boldsymbol{\nabla})\boldsymbol{\omega} - (\boldsymbol{\omega} \cdot \boldsymbol{\nabla})\mathbf{v} = R^{-1}\nabla^2\boldsymbol{\omega}, \tag{3}$$

where t is an appropriately scaled time and R is a suitable Reynolds number. The last equation is the vorticity transport equation.

Gatski[1] has recently reviewed the vorticity–velocity formulations in use and has identified three separate types of approach. All of them employ equation (3) to determine the vorticity, but they differ in the solution procedure for the velocity field. There are

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(i) methods in which the velocity field is obtained in integral form from the vorticity field, as in the work of Payne.[2] Wu[3] and Wu and Thompson[4]

(ii) methods where equations (1) and (2) are solved simultaneously or sequentially to determine $\mathbf{v}$ from a given approximation to $\boldsymbol{\omega}$, e.g. the work of Gatski *et al.*[5,6]

(iii) methods in which second-order equations are solved to determine the velocity components. For example, it is readily found from equations (1) and (2) that

$$\nabla^2 \mathbf{v} = -\text{curl}\, \boldsymbol{\omega}, \tag{4}$$

which may be solved for $\mathbf{v}$ for a given $\boldsymbol{\omega}$; early investigations of this type were carried out by Fasel[7–9] (see also the reference to the work of Fasel and co-workers in Reference 1); a similar method was given by Cook.[10]

In the present paper we discuss a method which does not strictly belong to any of the above classes but is in fact a combination of classes (ii) and (iii). We consider the problem of steady two-dimensional flow past a cylinder in which certain special features are apparent, notably concerned with the boundary conditions at large distance. In two dimensions there are only two velocity components and equation (3) reduces to a single scalar equation for the non-zero vorticity component. The present technique solves the vorticity transport equation by boundary value methods, assuming steady state flow. One of the velocity components is also determined by boundary value techniques from a second-order equation as in class (iii) above, but the other component is found by integrating the continuity equation (1) step-by-step outward from the cylinder surface. Using this method for the second velocity component, it is possible to obtain a solution which tends uniformly to the correct condition at large distance.

The method is described for a cylinder of arbitrary cross-section mapped on to a semi-infinite strip by a suitable conformal transformation. Illustrative results are then computed for the case of a circular cylinder, using fourth-order finite difference methods to approximate the equations and considering Reynolds numbers in the range $10 \leqslant R \leqslant 100$. They are compared with the results of recent calculations by Dennis and Hudson[11] and other workers and found to be in good agreement.

The formulation was first used by Hudson[12] for computing the flow past a sphere, although details of the method have not previously been published.

2. BASIC EQUATIONS

In general we apply a transformation of the form

$$x + \mathrm{i}y = F(\xi + \mathrm{i}\eta), \tag{5}$$

which places the cylinder at $\xi = \xi_0$. The transformation is assumed to be of Joukowski type, in which the flow at large distances from the cylinder remains the same after transformation. Thus, if the cylinder is in a uniform stream parallel to the x-axis, we may assume

$$x \sim k\mathrm{e}^{\xi} \cos(\eta + \alpha), \qquad y \sim k\mathrm{e}^{\xi} \sin(\eta + \alpha) \tag{6}$$

as $\xi \to \infty$ and the flow reduces to a uniform stream at an angle α with the positive x-direction.

The metric of the transformation is such that the equation of continuity (1) becomes

$$\frac{\partial}{\partial \xi}(Hu) + \frac{\partial}{\partial \eta}(Hv) = 0, \tag{7}$$

where

$$H(\xi, \eta) = \left[\left(\frac{\partial x}{\partial \xi}\right)^2 + \left(\frac{\partial x}{\partial \eta}\right)^2\right]^{1/2} = \left[\left(\frac{\partial y}{\partial \xi}\right)^2 + \left(\frac{\partial y}{\partial \eta}\right)^2\right]^{1/2}, \tag{8}$$

and equation (2), which defines the vorticity, becomes

$$\zeta = \frac{1}{H^2}\left(\frac{\partial}{\partial \xi}(Hv) - \frac{\partial}{\partial \eta}(Hu)\right). \tag{9}$$

Using equation (7) to eliminate either u or v from (9) leads to

$$\frac{\partial^2}{\partial \xi^2}(Hv) + \frac{\partial^2}{\partial \eta^2}(Hv) = \frac{\partial}{\partial \xi}(H^2\zeta) \tag{10}$$

and

$$\frac{\partial^2}{\partial \xi^2}(Hu) + \frac{\partial^2}{\partial \eta^2}(Hu) = -\frac{\partial}{\partial \eta}(H^2\zeta) \tag{11}$$

respectively. Finally, for two-dimensional motion there is a streamfunction ψ which satisfies

$$u = \frac{1}{H}\frac{\partial \psi}{\partial \eta}, \qquad v = -\frac{1}{H}\frac{\partial \psi}{\partial \xi} \tag{12}$$

and for steady motion the vorticity transport equation (3) becomes

$$\frac{\partial^2 \zeta}{\partial \xi^2} + \frac{\partial^2 \zeta}{\partial \eta^2} = \frac{R}{2}\left(Hu\frac{\partial \zeta}{\partial \xi} + Hv\frac{\partial \zeta}{\partial \eta}\right). \tag{13}$$

Given suitable boundary conditions, equations (10), (11) and (13) could of course be solved simultaneously to give u, v and ζ. One possible objection to this approach is that only a differentiated form of the continuity equation is used rather than the equation itself.

To overcome this objection, we propose that equation (7) be used to obtain u rather than equation (11). Another advantage of this approach is that, given v, it is possible to set up a stable step-by-step method to integrate (7) in the ξ-direction, which leads uniformly to the necessary free stream condition for u. The latter feature makes the proposed formulation particularly appropriate for flow past a cylinder. In addition, as shown in the next section, satisfactory boundary conditions for v at large distance can also be derived.

3. METHOD OF SOLUTION

Using finite difference methods, equation (10) is solved as a boundary value problem at all points of the domain

$$0 < \eta < 2\pi, \qquad \xi_0 < \xi < \xi_m, \tag{14}$$

where ξ_m is some large enough value of ξ at which the conditions as $\xi \to \infty$ may be approximated.

In general the boundary conditions for v are obtained from the periodic condition

$$v(\xi, \theta) = v(\xi, \theta + 2\pi), \tag{15}$$

with

$$v = 0 \quad \text{on } \xi = \xi_0, \qquad v = v(\xi_m, \theta) \quad \text{on } \xi = \xi_m. \tag{16}$$

For the type of transformation used, the conditions as $\xi \to \infty$ take the form

$$u \to k e^{\xi} \cos \eta, \qquad v \to k e^{\xi} \sin \eta, \tag{17}$$

where k is a constant which depends upon the shape of the cylinder. Thus one possible condition for v on $\xi=\xi_m$ is simply

$$v = k e^{\xi_m} \sin\eta. \tag{18}$$

However, a better condition can be obtained as follows. It is convenient to introduce the perturbed velocity v^*, where

$$v = v^* + k e^{\xi} \sin\eta, \tag{19}$$

so that $v^* \to 0$ as $\xi \to \infty$. Note that v^* also satisfies equation (10). From the asymptotic theory of Oseen we known that a well-defined wake in which the vorticity is significant exists behind the cylinder and extends downstream. In the Cartesian plane this wake is expanding and with a roughly parabolic boundary,[13] but in the present co-ordinate system it corresponds to a wake which narrows near the co-ordinate curves $\eta=0,\ 2\pi$ with a breadth proportional to $R^{-1/2}e^{-1/2\xi}$. Therefore, in this region, derivatives with respect to η are of the order $R^{1/2}e^{1/2\xi}$ compared with those with respect to ξ. Thus, if we substitute (19) into (10), we can neglect the second derivative of v^* with respect to ξ as $\xi \to \infty$, giving

$$\frac{\partial^2}{\partial\eta^2}(Hv_m^*) = \frac{\partial}{\partial\xi}(H^2\zeta_m) \tag{20}$$

at $\xi=\xi_m$. Integrating this equation using the conditions

$$v_m^* = 0 \quad \text{at } \eta = 0,\ \pi \tag{21}$$

gives the required expression for v_m^* to be used as a boundary condition in the solution of equation (10). Thus the boundary condition for v^* on $\xi=\xi_m$ comes from the equation itself by virtue of the fact that the equation becomes dominant in the η-direction.

Equation (13) is also solved by finite difference methods in the region defined by (14). Boundary conditions for equation (13) come from a generalization of the procedure used by Dennis and Chang[14] in solving the problem of flow past a circular cylinder. It is based on the Oseen linearization of (13) using the components (17) for u and v. The theory is essentially the same and so will not be repeated here. On the surface of the cylinder the vorticity is evaluated using the global procedures described by Dennis and Quartapelle.[15]

A step-by-step solution of equation (7) can be obtained if it is expressed in the form

$$(Hu)_{\xi_p} - (Hu)_{\xi_0} = -\int_{\xi_0}^{\xi_p} \frac{\partial}{\partial\eta}(Hv)\,d\xi, \quad p = 1, 2, \ldots, m. \tag{22}$$

Then, assuming that an approximation to v has been obtained throughout the field by solving equation (10), we may determine $u(\xi,\eta)$ from $u(\xi_0,\eta)$ by approximating the right-hand side of equation (22) using appropriate differentiation and integration formulae. Thus $u(\xi,\eta)$ is obtained at all grid points by integration along each set of grid points for a constant value of η, repeating the procedure for each value of η. It may be noted that for the Joukowski type of transformation the integration (22) is stable as $\xi \to \infty$, because $H \sim e^{\xi}$ as $\xi \to \infty$ and so $H(\xi_0,\eta)/H(\xi_p,\eta) \sim e^{-(\xi_p-\xi_0)} < 1$. Also, for a circular cylinder, $H=e^{\xi}$ for all ξ, and for an elliptic cylinder inclined at an angle α to the stream, $H=\{\cosh(2\xi) - \cos 2(\eta+\alpha)\}^{1/2}$. Hence $\partial H/\partial\xi > 0$ for all ξ and so the integration (22) is always stable.

The above procedures can be implemented in several ways. For example, one iteration of equation (10) could be followed by the integration (22) and the values of u and v obtained used in a single iteration of equation (13). This process would be repeated with the boundary values updated until convergence to the required accuracy was obtained.

4. FINITE DIFFERENCE APPROXIMATION

Equations (10) and (13) have the form

$$\frac{\partial^2\phi}{\partial\xi^2}+\frac{\partial^2\phi}{\partial\eta^2}-a(\xi,\eta)\frac{\partial\phi}{\partial\xi}-b(\xi,\eta)\frac{\partial\phi}{\partial\eta}-c(\xi,\eta)\phi=R(\xi,\eta). \tag{23}$$

Dennis and Hudson[16] have proposed a fourth-order finite difference method for a similar equation in which $c(\xi,\eta)=0$. This method can easily be extended to the present problem, giving a nine-point formula

$$\sum_{n=1}^{8}d_n\phi_n-d_0\phi_0+E_0=0,$$

where the subscripts 0–8 correspond respectively to the points (ξ_0,η_0), (ξ_0+h,η_0), (ξ_0,η_0+h), (ξ_0-h,η_0), (ξ_0,η_0-h), (ξ_0+h,η_0+h), (ξ_0-h,η_0+h), (ξ_0-h,η_0-h) and (ξ_0+h,η_0-h) of a square grid of side h and

$$d_0=40+2h^2(a_0^2+b_0^2)-4h^2\left[\left(\frac{\partial a}{\partial\xi}\right)_0+\left(\frac{\partial b}{\partial\eta}\right)_0\right]+8h^2c_0,$$

$$d_1=8-4ha_0+h^2\left[a_0^2-2\left(\frac{\partial a}{\partial\xi}\right)_0\right]+\frac{h^3}{2}\left[a_0\left(\frac{\partial a}{\partial\xi}\right)_0+b_0\left(\frac{\partial a}{\partial\eta}\right)_0-(\nabla^2a)_0\right]-h^2\left(1-\frac{h}{2}a_0\right)c_1,$$

$$d_2=8-4hb_0+h^2\left[b_0^2-2\left(\frac{\partial b}{\partial\eta}\right)_0\right]+\frac{h^3}{2}\left[a_0\left(\frac{\partial b}{\partial\xi}\right)_0+b_0\left(\frac{\partial b}{\partial\eta}\right)_0-(\nabla^2b)_0\right]-h^2\left(1-\frac{h}{2}b_0\right)c_2,$$

$$d_3=8+4ha_0+h^2\left[a_0^2-2\left(\frac{\partial a}{\partial\xi}\right)_0\right]-\frac{h^3}{2}\left[a_0\left(\frac{\partial a}{\partial\xi}\right)_0+b_0\left(\frac{\partial a}{\partial\eta}\right)_0-(\nabla^2a)_0\right]-h^2\left(1+\frac{h}{2}a_0\right)c_3,$$

$$d_4=8+4hb_0+h^2\left[b_0^2-2\left(\frac{\partial b}{\partial\eta}\right)_0\right]-\frac{h^3}{2}\left[a_0\left(\frac{\partial b}{\partial\xi}\right)_0+b_0\left(\frac{\partial b}{\partial\eta}\right)_0-(\nabla^2b)_0\right]-h^2\left(1+\frac{h}{2}b_0\right)c_4,$$

$$d_5=2-h(a_0+b_0)+\frac{h^2}{2}a_0b_0-\frac{h^2}{2}\left[\left(\frac{\partial b}{\partial\xi}\right)_0+\left(\frac{\partial a}{\partial\eta}\right)_0\right],$$

$$d_6=2+h(a_0-b_0)-\frac{h^2}{2}a_0b_0+\frac{h^2}{2}\left[\left(\frac{\partial b}{\partial\xi}\right)_0+\left(\frac{\partial a}{\partial\eta}\right)_0\right],$$

$$d_7=2+h(a_0+b_0)+\frac{h^2}{2}a_0b_0-\frac{h^2}{2}\left[\left(\frac{\partial b}{\partial\xi}\right)_0+\left(\frac{\partial a}{\partial\eta}\right)_0\right],$$

$$d_8=2-h(a_0-b_0)-\frac{h^2}{2}a_0b_0+\frac{h^2}{2}\left[\left(\frac{\partial b}{\partial\xi}\right)_0+\left(\frac{\partial a}{\partial\eta}\right)_0\right],$$

$$E_0=-h^2\left[8R_0+\left(1-\frac{h}{2}a_0\right)R_1+\left(1-\frac{h}{2}b_0\right)R_2+\left(1+\frac{h}{2}a_0\right)R_3+\left(1+\frac{h}{2}b_0\right)R_4\right],\ \nabla^2=\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}.$$

This finite difference scheme was used to solve equations (10) and (13) in the illustrative example in the next section. To preserve the h^4-accuracy, we used suitably accurate expressions for the derivative

$\partial(Hv)/\partial\eta = r(\xi, \eta)$ in equation (22). For the first step of the integration ($p = 1$) we used the fourth-order formula.

$$u(h, \eta) = \alpha u(0, \eta) - h[9r(0, \eta) + 19r(h, \eta) - 5r(2h, \eta) + r(3h, \eta)]/24hH(h, \eta), \tag{24}$$

where

$$\alpha = H(0, \eta)/H(h, \eta),$$

and Simpson's rule for $p = 2, 3, \ldots, m$.

5. CALCULATED RESULTS

The main objective of the present paper is to present a version of the vorticity–velocity formulation which is suitable for calculating flow past a cylinder. As a numerical example we consider the case of a circular cylinder for which good comparison results are known. For a circular cylinder the function $F(\xi + i\eta)$ in equation (5) is $\exp(\xi + i\eta)$ and $H = e^{\xi}$. Equations (7), (10) and (13) then become

$$\frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \eta} + u = 0, \tag{25}$$

$$\frac{\partial^2 v}{\partial \xi^2} + \frac{\partial^2 v}{\partial \eta^2} + 2\frac{\partial v}{\partial \xi} + v = e^{\xi}\left(2\zeta + \frac{\partial \zeta}{\partial \xi}\right) \tag{26}$$

and

$$\frac{\partial^2 \zeta}{\partial \xi^2} + \frac{\partial^2 \zeta}{\partial \eta^2} - \frac{1}{2}Re^{\xi}u\frac{\partial \zeta}{\partial \xi} - \frac{1}{2}Re^{\xi}v\frac{\partial \zeta}{\partial \eta} = 0 \tag{27}$$

respectively, where R is the Reynolds number based on the diameter of the cylinder. These equations were solved for $R = 10$, 40 and 100 in the manner described in Section 3, using the finite difference approximations given in Section 4 for equations (26) and (27) and the formula

$$u(\xi_p) = e^{-\xi_p}u(0) - e^{-\xi_p}\int_0^{\xi_p} e^{\xi}\frac{\partial v}{\partial \eta}d\xi, \quad p = 1, 2, \ldots, m, \tag{28}$$

corresponding to equations (22) to integrate (25). The conditions on v at large distance described in Section 3 become

$$v(\xi_m, \eta) = \int_0^{\eta} \phi(\theta)d\theta - \frac{\eta}{\pi}\int_0^{\pi} \phi(\theta)d\theta, \tag{29}$$

where

$$\phi(\theta) = -\int_0^{\theta} e^{\xi}\left(2\zeta + \frac{\partial \zeta}{\partial \xi}\right)d\theta. \tag{30}$$

For each value of R, solutions were obtained for various step lengths and with $\xi_m \geqslant \pi$. In each case convergence was assumed when

$$\max\left\{\sum_{i,j} |\zeta_{i,j}^{(k+1)} - \zeta_{i,j}^{(k)}|, \sum |u_{i,j}^{(k+1)} - u_{i,j}^{(k)}|, \sum |v_{i,j}^{(k+1)} - v_{i,j}^{(k)}|\right\} < \varepsilon,$$

where, typically, $\varepsilon = 0{\cdot}001$, k is an iteration count and the summations cover all the nodes in the domain defined in (14).

Table I. Calculated properties of numerical solutions and comparison with other results

Reference	R	h	C_F	C_P	C_D	$P(0)$	$P(\pi)$
Dennis and Hudson[11]	10	$\pi/40$	1·217	1·555	2·772	−0·684	1·478
Present results			1·216	1·556	2·772	−0·687	1·478
Dennis and Hudson[11]		$\pi/60$	1·213	1·550	2·763	−0·680	1·477
Present results			1·223	1·561	2·784	−0·691	1·479
Dennis and Hudson[11]	40	$\pi/40$	0·525	0·977	1·502	−0·47	1·143
Present results			0·523	0·985	1·508	−0·49	1·143
Dennis and Hudson[11]		$\pi/60$	0·523	0·985	1·508	−0·43	1·142
Present results			0·519	0·988	1·507	−0·49	1·142
Fornberg[17]		—	—	—	1·498	−0·46	1·140
Dennis and Hudson[11]	100	$\pi/40$	0·281	0·687	0·968	−0·28	1·060
Present results			0·284	0·727	1·011	−0·35	1·060
Dennis and Hudson[11]		$\pi/60$	0·287	0·766	1·053	−0·38	1·060
Present results			0·287	0·775	1·062	−0·41	1·060
Fornberg[17]		—	—	—	1·050	−0·34	1·064

Illustrative results for the drag coefficients (C_F and C_P) and the pressure on the cylinder surface at $\eta = 0$, π are given in Table I, where they are compared with other results. Those of Dennis and Hudson[11] were obtained by solving the stream function and vorticity equations by fourth-order finite difference methods with the same vorticity boundary conditions as in the present study. Those of Fornberg[17] are included as an independent check. Bearing in mind the difficulties associated with calculating C_P and $p(0)$ accurately, the present results are in satisfactory agreement and illustrate that the proposed formulation is indeed suitable for calculating the flow past a cylinder.

As mentioned in Section 2, one theoretical advantage of calculating u from equation (7) rather than from equation (11) is that the continuity equation itself will be satisfied rather than a differentiated form of it. To test this theory, an additional solution for $R = 40$ with $h = \pi/40$ was computed using the appropriate form of equation (11), namely

$$\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2} + 2\frac{\partial u}{\partial \xi} + u - e^{\xi}\frac{\partial \zeta}{\partial \eta} = 0, \tag{31}$$

in place of equation (25). Note that equation (31) has the same form as equation (23) and so the fourth-order method of Section 4 was again applied. In this case, conditions for u at large distance were obtained from a simple modification of equation (29), namely

$$u(\xi_m) = \mathrm{e}^{-\xi_m} u(\xi_p) - \mathrm{e}^{-\xi_m}\int_{\xi_p}^{\xi_m} \mathrm{e}^{\xi}\frac{\partial v}{\partial \eta}\mathrm{d}\xi, \quad p < m. \tag{32}$$

The value of p in this equation is somewhat arbitrary, but setting $p = m - 2$ allows the use of Simpson's rule to estimate the integral accurately.

From each solution at $R = 40$ the quantity

$$C = \sum_{i,j}\left|\frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \eta} + u\right|_{i,j}$$

was calculated, with the summation extending to all points $(\xi_i, \ \eta_j)$ in the region described by (14). It was found that $C = 0{\cdot}19$ when u was obtained from (25) but $C = 9{\cdot}72$ when u was obtained from (31)

Table II. See text

u calculated from	Surface vorticity calculated from	C (continuity)	Number of iterations	CPU time (s)
Equation (7)	Dennis and Quartapelle[15]	0.19	935	496
Equation (11)	Dennis and Quartapelle[15]	9.72	2771	1377
Equation (7)	Equation (9)	0.20	1047	207
Equation (11)	Equation (9)	1.23	2841	479

and so the theory is supported. It is interesting to note that the total numbers of iterations required for convergence of the two solutions were 935 and 2771 respectively, requiring 496 and 1377 s of CPU time (on a 486/50 PC) respectively. Thus the proposed formulation is much the more efficient of the two considered.

As an experiment the two computer programmes used above were modified so that the vorticity on the cylinder surface was computed using an h^4-accurate finite difference approximation to equation (9) instead of the integral method of Dennis and Quartapelle.[15] Again the proposed formulation produced a lower value of C (0·20 versus 1·23) and required fewer iterations (1047 versus 2841) and much less CPU time (207 versus 479 s). For convenience these values are reproduced in Table II. It may be noted from this table that the computational efficiency of the integral method of calculating the boundary vorticity is, as may be expected, less than that of the derivative method. Nevertheless, it is extremely valuable to have available both local and global methods.

6. CONCLUSIONS

A vorticity–velocity formulation for computing the flow past a cylinder is proposed in which one of the velocity components is obtained by integrating the equation of continuity rather than from a second-order equation related to it. Calculations for the case of a circular cylinder suggest that the proposed formulation gives satisfactory results and is more than twice as efficient as the alternative method considered. It also produced solutions which satisfy the equation of continuity more accurately.

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